THE ABSORPTION SPECTRUM OF THE NOVAE

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One of the most remarkable features of the spectrum of novae or temporary stars is the presence at a certain stage in their development of absorption lines displaced greatly toward the violet end of the spectrum. In the case of the elements hydrogen and helium they appear as strong lines forming sharp boundaries to the broad emission bands which are the principal characteristic of the spectrum of such stars. Occasionally they are double and are subject to great variations in character and intensity. In addition to the lines of hydrogen and helium a large number of absorption lines has been observed in the spectra of the three brightest novae of recent years, Nova Persei of 1901, Nova Geminorum of 1912, and Nova Aquilae of 1918. The identification of many of these lines and some features of their displacements form the object of this communication.

Nova Aquilae was first observed at Mount Wilson on June 8. At this time the spectrum was essentially continuous, with a few broad and hazy absorption bands superposed upon it. Most of these were due to hydrogen and helium and were displaced over 20 angstroms toward the violet. Two nights later the entire spectrum had changed: a large number of comparatively narrow absorption lines had appeared and the hydrogen and helium lines had become double. This spectrum had so many points of similarity to that of certain stars that a comparison was instituted with a photograph of α Cygni, a star of early type with exceptionally strong enhanced lines. It then appeared that a considerable number of lines could be identified as common to both spectra provided a displacement of about -23 angstroms at λ 4500 were assumed in the case of Nova Aquilae. A large proportion of these lines are enhanced but some of the stronger arc lines are also represented. In the region of spectrum between λ 3900 and Hβ ninety lines have been identified with considerable certainty, and the origin of many others may be assigned with only slightly less probability.

A similar comparison with the lines measured in the spectrum of Nova Geminorum of 1912[1] shows clearly the presence of many of these lines in the latter star as well. The displacement, however, is only about one-half as great as in the case of Nova Aquilae. To illustrate the character of the lines identified and the displacements involved the results for a portion of the spectrum between λ 4500 and λ 4600 are given in table 1.

The lines in the spectrum of Nova Geminorum are much more hazy and ill-defined than those in Nova Aquilae and the agreement is much less satisfactory. This is due in part to the fact that the earliest photographs of the spectrum of Nova Geminorum were obtained at a longer interval after the discovery of the star than in the case of Nova Aquilae, and that during this interval the absorption lines had begun to grow faint and disappear. About forty lines common to the spectra of these two stars have been identified in the region between Hδ and Hβ.

TABLE 1

WAVE-LENGTH			DISPLACEMENT		ELEMENT	
Sun	Nova Aquilae (June 12)	Nova Geminorum	Nova Aquilae A	Nova Geminorum A		
4501.4	4478.2	4489.3	−23.2	−12.1	Enhanced	Ti
4508.5	4485.3	4495.5	23.2	13.0	Enhanced	Fe
4515.5	4492.0	4505.2	23.5	10.3	Enhanced	Fe
4520.4	4497.1	4508.1	23.3	12.3	Enhanced	Fe
4522.8	4499.2	4511.9	23.6	10.9	Enhanced	Fe
4528.8	4505.3	4518.4	23.3	10.4	Arc line	Fe
4534.1	4510.5	4522.8	23.6	11.3	Enhanced	Ti
4549.8	4526.1	4537.6	23.7	12.2	Enhanced Fe, Ti	
4556.1	4531.7	4543.8	24.4	12.3	Enhanced	Fe
4558.8	4535.0	4547.0	23.8	11.8	Enhanced	Cr
4563.9	4540.1	4551.6	−23.8	−12.3	Enhanced	Ti

It is known from the investigations of Campbell and Wright on the spectrum of Nova Persei of 1901[2] that the displacements of the absorption lines of hydrogen were directly proportional to wave-length in this star, and the same result was found for Nova Geminorum by other observers. The identification of the metallic lines in the case of Nova Aquilae and Nova Geminorum makes a study of their displacements of especial interest in this regard. For this purpose the average displacement and the average wave-length have been formed for groups of lines distributed throughout the spectrum. In the less refrangible region longer intervals have been included because of the smaller number of lines available and the larger range of error in wave-length. The results are given in table 2.

The values for Nova Persei are by Campbell and Wright[2] and those for Nova Aurigae of 1892 are by Campbell and Vogel.[3]

These results may be represented by the following equations in which $\Delta\lambda$ is the displacement. In the case of Nova Aurigae the results are too fragmentary to allow of an accurate determination of the coefficient of λ.

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Displacement
at λ 4500

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Nova Aquilae:	$\Delta\lambda = 0.00513\ \lambda$.................................	-23.1
Nova Persei:	$\Delta\lambda = 0.00510\ \lambda$.................................	-22.9
Nova Geminorum:	$\Delta\lambda = 0.00257\ \lambda$.................................	-11.5
Nova Aurigae:	$\Delta\lambda = 0.0025\ \lambda$.................................	-11.4

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TABLE 2

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MEAN λ	MEAN $\Delta\lambda$	NUMBER OF LINES	O − C	MEAN λ	MEAN $\Delta\lambda$	NUMBER OF LINES	O − C
Nova Aquilae				*Nova Geminorum*			
4020	20.8	5	+0.2	4220	10.6	7	−0.2
4155	21.5	4	+0.2	4350	10.9	7	−0.3
4260	22.0	7	+0.1	4445	11.2	5	−0.2
4345	22.5	11	+0.2	4540	11.8	13	+0.2
4450	22.9	7	+0.1	4935	12.8	3	+0.1
4540	23.6	13	+0.3	5896	15.3	1	+0.2
4630	24.0	2	+0.2	6563	17.9	1	+1.1
4820	24.6	4	−0.1	*Nova Persei (H lines)*			
4990	25.5	3	−0.1				
5230	27.2	8	+0.4	3890	19.3	1	−0.5
5350	27.6	8	+0.2	3970	20.0	1	−0.2
5510	28.1	5	−0.2	4102	20.5	1	−0.4
5670	27.8	4	−1.3	4341	22.2	1	+0.1
5890	29.4	5	−0.8	4862	25.7	1	+0.9
6390	32.7	10	−0.1	*Nova Aurigae (H lines)*			
				4102	10.3	1	0.0
				4341	11.4	1	+0.5
				4862	11.9	1	−0.4

The differences, observed minus computed, are given under O − C in table 2. The agreement is close except in the less refrangible part of the spectrum where the determinations of wave-length are least accurate.

We find, accordingly, the surprising result that the displacements of the lines in all of these stars are directly proportional to wave-length and divide themselves into two pairs of equal amount. Of these the first pair of stars has exactly twice the displacement of the second, and it is perhaps a significant fact that Nova Aquilae and Nova Persei were much brighter stars apparently, and probably intrinsically as well, than Nova Geminorum and Nova Aurigae.

A very peculiar phenomenon in the case of Nova Aquilae is the progressive increase in the values of the displacements of the absorption lines at successive dates. Thus in the region between λ 4250 and λ 4600 we find for the dates June 10 to June 15:

June 10	June 11	June 12	June 13	June 15
−21.7(30)	−22.4(30)	−23.0(26)	−23.7(19)	−24.6(11)

During this period the average daily increase in displacement is slightly over 0.5 angstrom. The decrease in the number of lines (given in parenthesis) shows the gradual disappearance of the absorption spectrum within this time.

Reference has already been made to the double character of the hydrogen and helium lines during the interval when the absorption spectrum of Nova Aquilae was prominent. A similar characteristic was found in the case of Nova Geminorum. The less refrangible of these components shows displacements which are in close agreement with those of the absorption lines due to iron and other elements which have been identified. An investigation of the more refrangible component shows that its displacements may also be represented as a direct function of the wave-length. We find for the two stars the relations:

Nova Aquilae: $\Delta\lambda = 0.0075\ \lambda$:   Nova Geminorum: $\Delta\lambda = 0.0048\lambda$

The differences as computed from these equations are as follows:

	NOVA AQUILAE (JUNE 12)			NOVA GEMINORUM		
	Observed	Computed	O − C	Observed	Computed	O − C
H...............	30.7	30.8	−0.1	19.4	19.7	−0.3
H...............	31.7	32.6	−0.9	20.6	20.8	−0.2
Hβ...............	37.6	36.5	+1.1	23.7	23.3	+0.4
Hα...............	49.0	49.2	−0.2			

At λ 4500 we find the displacements

Nova Aquilae; −33.8   Nova Geminorum; −21.6

In the case of Nova Geminorum, therefore, the displacement of this component of the hydrogen lines is almost exactly twice that of the less refrangible component and the numerous metallic lines, while in Nova Aquilae it is one and one-half times as great. In other words 11, 22 and 33 angstroms, in the harmonic ratio 1, 2 and 3, represent very closely all the displacements found among the absorption lines in the spectra of these four stars with the exception of the four narrow nearly undisplaced lines of calcium and sodium from which the radial velocities of the stars may be derived. These lines, like the broad emission bands of hydrogen and helium, may perhaps be considered as belonging to the stars themselves, while the greatly displaced absorption lines originate in an outer envelope possibly detached from the body of the stars.

It is certain that no adequate explanation has been offered as yet to account for these immense displacements. Were the hydrogen and helium lines alone involved it is possible that some of the complex phenomena of self-reversal under conditions of marked variations in density and pressure might, at least in part, be responsible. The presence, however, of a great number of com-

paratively narrow single absorption lines unaccompanied by emission bands cannot be explained in this way. Their character, as well as the law of variation of displacement with wave-length, also precludes the agency of pressure (assuming that negative displacements can be produced by pressure) and the Zeeman effect. No dependence of anomalous dispersion upon wave-length is known, even if it were adequate to produce lines of such a character with such displacements.

In some respects the Doppler effect accounts most nearly for the results observed. Motion in the line of sight would produce displacements directly proportional to wave-length and might leave the spectral lines well-defined. The velocities involved, -750 km. in the case of Nova Geminorum and Nova Aurigae, and -1500 km. for Nova Aquilae and Nova Persei, though large, are of the order found for some spiral nebulae, and not many times greater than those observed in some of the solar prominences. If the more refrangible component of the hydrogen, helium and calcium lines is considered, the velocities amount to -1500 and -2200 km. respectively. These values are for motion in the line of sight. If a generalized form of the Doppler principle is used a part or the whole of the effect might be referred to a rapid change in the thickness of stratum of gas producing the absorption, or in its refractive index. This hypothesis was suggested by W. Michelson[4] to account for the high velocities observed in solar prominences, and by Paddock[5] in the case of Nova Aquilae. Some objections to this view are the rate of change in refractive index or in thickness of the gas which would be required, especially since a decrease of index would be needed to produce displacements toward shorter wave-lengths; the harmonic relationship found among the displacements; and the relatively narrow character of the absorption lines.

The last two objections might also apply to motion in the line of sight. The suggestion, however, may be made that the absorption lines are produced in a shell of gas which is moving radially outward from the star with a high velocity. If the size of this shell is large as compared with that of the body of the star it is evident that an area of the shell only equal to that of the star would be seen in projection against the latter and would give absorption lines, and that all of the gas within this area would have large components of velocity toward the observer. This would result in comparatively narrow absorption lines. The remainder of the shell of gas would give an emission spectrum, and the combination of the widely different velocities would result in very broad bright bands with their centers nearly undisplaced. This is in accordance with observations. With these high velocities the interval of two days between the outburst of the star and the appearance of the prominent absorption spectrum would be sufficient for the gas to reach a great distance from the surface of the star. The hypothesis would, however, leave unexplained the apparent acceleration of motion during the period of observation of the absorption spectrum; and the nearly constant character of the emission bands after the disappearance of the absorption spectrum would point rather to their origin in the star itself.

The possibility of some form of dissociation is suggested by the harmonic relationship between the displacements of the components of the hydrogen lines, and the early appearance of the nebular lines in the spectra of these stars adds interest to considerations of this nature.

[1] Adams, W. S., and Kohlschütter, A., *Mt. Wilson Contr.* No. 62, *Astroph. J., Chicago, Ill.*, **36**, 1912, (293–321).

[2] Campbell, W. W., and Wright, W. H., *Lick Obs. Bul.* No. 8, *Berkeley, Cal.*, 1901.

[3] Scheiner, J., *Astronomical Spectroscopy* (Frost), *Boston, Mass.*, 1894, p. 290.

[4] Michelson, W., *Astroph. J., Chicago, Ill.*, **13**, 1901, (192–198).

[5] Paddock, G. F., *Pub. Astr. Soc. Pac., San Francisco, Cal.*, **30**, 1918, (244–249), p. 249.

ON JACOBI'S EXTENSION OF THE CONTINUED FRACTION ALGORITHM

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It has been known since Lagrange[1] that the regular continued fraction which represents a quadratic surd becomes periodic after a finite number of non-periodic partial quotients, and conversely, a regular continued fraction which becomes periodic after a finite number of non-periodic partial quotients is one root of a quadratic equation with rational coefficients. It is useless, therefore, to look for periodicity in regular continued fractions which represent cubic and higher irrationalities. To meet this difficulty Jacobi[2] undertook to extend the continued fraction algorithm as follows:

In the case of the ordinary continued fraction we are concerned with two series of numbers, A_n, B_n, (the numerators and denominators of the successive convergents) which are given by the recursion formulae

$$A_n = q_n A_{n-1} + A_{n-2}, \quad B_n = q_n B_{n-1} + B_{n-2},$$

with the initial values $A_0 = 0$, $A_{-1} = 1$, $B_0 = 1$, $B_{-1} = 0$. Jacobi considers three series of numbers, A_n, B_n, C_n, which are given by the recursion formulae

$$A_n = p_n A_{n-1} + q_n A_{n-2} + A_{n-3},$$
$$B_n = p_n B_{n-1} + q_n B_{n-2} + B_{n-3},$$
$$C_n = p_n C_{n-1} + q_n C_{n-2} + C_{n-3},$$

with the initial values:

$$(A_{-2}, A_{-1}, A_0) = (1, 0, 0), \quad (B_{-2}, B_{-1}, B) = (0, 1, 0),$$
$$(C_{-2}, C_{-1}, C_0) = (0, 0, 1),$$

Jacobi then chose the coefficients of the expansion, p_n, q_n, so that $A_n : B_n : C_n$ should approximate $1 : \theta : a + b\theta + c\theta^2$ where θ is the real root of a cubic equa-